

**Pre TPS**  
**Dimensions and Units Tutorial**

1. Work out the dimensions (in terms of M, L and T) of the following groups of familiar parameters:

a.  $m \frac{V^2}{R}$

b.  $mR\omega^2$

c.  $\sqrt{b^2 + c^2}$  (b and c both lengths)

d.  $2\pi \sqrt{\frac{l}{g}}$

e.  $\frac{1}{3}\pi R^2 h$

f.  $mc^2$

(m = mass; V = velocity; R = radius;  $\omega$  = angular velocity; l = length; g = acc. due to gravity; h = height; c = velocity of light)

2. The force of gravitational attraction between the earth and the moon is given by the equation:

$$F = \frac{GMm}{r^2}$$

where M and m are the two masses, r the distance between mass-centers, and G the gravitational constant. What are the dimensions of G? What are its units in a) the metric system, and b) the imperial system?

3. State whether or not the following equations are dimensionally consistent:

a.  $v = u + at$       b.  $v^2 = u^2 + 2ts$       c.  $\tan\theta = \frac{v_o \sin\theta}{v_o \cos\theta + V}$

d.  $s = ut + \frac{1}{2} at^2$       e.  $v_2^2 = \frac{2(P_1 - P_2)}{r} \frac{A_1}{A_1^2 - A_2^2}$       f.  $f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$

(v, u, v and V = velocities; s = distance; t = time; a = acceleration; P = pressure; A = area; f = frequency; K is Hooke's constant – force per unit deflection.)

4. Given that the gravitational force between two planetary objects is  $F = \frac{GMm}{r^2}$ , and that the centripetal force towards the centre of a circle is  $F = mr\omega^2$ , calculate, in kilometers, the height above the earth of a geo-stationary satellite.

( $M = 6 \times 10^{24}$  kgs;  $G = 6.67 \times 10^{-11}$  nt.m<sup>2</sup>/kg<sup>2</sup>)

5. Air at standard temperature and pressure flows down a straight pipe, of uniform cross-sectional area 2 sq. ft., at a rate of 1 slug/sec. A penstock valve is quickly closed bringing the flow instantaneously to rest. Assuming no losses, what is the peak rise in static pressure at the valve in lbs/sq.ft. and in lbs/sq. in. How many atmospheres is this pressure rise? (use  $P_{\text{tot}} = P_s + \frac{1}{2} \rho V^2$ )  $\rho = 2.37 \times 10^{-3}$  slugs/cu.ft.

Name: \_\_\_\_\_

Algebra Tutorial

1. Combine:

a.  $2x + (3x - 4y)$  \_\_\_\_\_

b.  $4x^2 + 5x - (3x - 7) + (-2x^2 + 3)$  \_\_\_\_\_

c.  $[(x + 2y) - (x + 3y)] - [(2x + 3y) - (-4x + 5t)]$   
\_\_\_\_\_

2. Add:

a.  $x^2 + 2x - 1 + 3x - 4 + 2x^2 + 5$  \_\_\_\_\_

b.  $7x + 3y^3 - 4xy$ ,  $3x - 2y^3 + 7xy$ ,  $2xy - 5x - 6y^3$   
\_\_\_\_\_

c.  $\theta + \alpha + 2\alpha - \theta + 3\theta + 4\alpha$  \_\_\_\_\_

3. Subtract:

a.  $2x^2 - 3xy + 5y^2$  from  $10x^2 - 2yx - 3y^2$   
\_\_\_\_\_

4. Remove brackets and simplify:

a.  $2(x^2 - 4x)$

\_\_\_\_\_

b.  $-a(2a + 3b)$

\_\_\_\_\_

c.  $2x[-4(3 + 2y) + (x + y + 1)]$

\_\_\_\_\_

d.  $2(t^3 + 1.4t^2 - 2.7t) - 4(0.5t^3 - t^2 + 1.3t)$

\_\_\_\_\_

5. Multiply and simplify:

a.  $(x + y)(x + 4)$

\_\_\_\_\_

b.  $(3xy)(2x^2y + 3y^2x + 3xy)$

\_\_\_\_\_

c.  $(x - y)(x^2 + y + 3)$

\_\_\_\_\_

d.  $(p + 6q)(p^2 + 2pq + q^2)$

\_\_\_\_\_

6. Divide:

a.  $(24x^4y^2z^3) \div (-3x^3y^4z)$

---

b.  $[x^2 + 2x^4 - 3x^3 + x - 2] \div [x^2 - 3x + 2]$

---

7. Factor:

a.  $x^2 + xy$

---

b.  $x^2 - y^2$

---

c.  $4x^2 - y^2$

---

d.  $x^2 - 7x + 6$

---

e.  $x^2 + 2xy - 8y^2$

---

f.  $6x^2y + 4y^2x + 2$

---

8. Simplify:

a.  $\frac{x^2 - xy}{x^2 - 3x}$

---

b.  $\frac{x^2 - y^2}{(x + y)^2}$

---

c.  $\frac{x^2 - 3x + 2}{2 - x}$

---

9. Express as a single fraction:

a.  $\frac{1}{x} + \frac{4}{y}$

---

b.  $\frac{4}{3xy} - \frac{5}{6yz}$

---

c.  $\frac{6}{x^2 - 6} + \frac{3x}{x^2 + 2}$

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Name: \_\_\_\_\_

Linear & Quadratic Equations Tutorial

1. Solve for  $x$  :

a.  $7x - 3 = 25$

\_\_\_\_\_

b.  $2x + 1 = 3x - 3$

\_\_\_\_\_

c.  $3(x + 7) - 2(x + 13) = 0$

\_\_\_\_\_

d.  $\frac{x-2}{x+2} = \frac{x-4}{x+4}$

\_\_\_\_\_

2. Solve for  $x$  and  $y$

a.  $3x + 6y = 11$   
 $14x - y = 3$

\_\_\_\_\_

b.  $-3y + 2x = 2$   
 $3x + 5y = 41$

\_\_\_\_\_



c.  $3x - 1 = -y + 7$   
 $x + 3y = 0$

---

3. Solve for  $x$ ,  $y$ , and  $z$

$$x + y + z = 0$$

a.  $3x - 3y - 3z = -12$

$$x - y + 2z = -7$$

---

$$2x - y - 3z = -11$$

b.  $x - 2y - z = -15$

$$3x + 3y + z = 26$$

---

4. Solve for  $x$  by factorization:

a.  $x^2 + 3x + 2 = 0$

---

b.  $x^2 + 8x + 15 = 0$

---

c.  $x^2 - 6x + 9 = 11$

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5. Solve for  $x$  using the standard quadratic formula:

a.  $3x^2 - 5x + 1 = 0$

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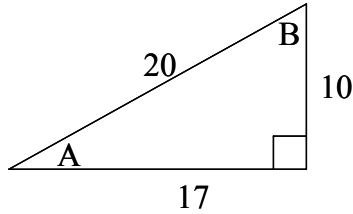
b.  $2x^2 - 6x + 3 = 0$

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Name: \_\_\_\_\_

**Trigonometry Tutorial**

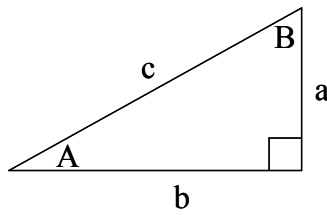
1. Given:



Find:  $\sin A$  \_\_\_\_\_  
 $\cos A$  \_\_\_\_\_  
 $\tan A$  \_\_\_\_\_

2. Given:  $\sin A = 2/5$

$c = 5$



Find:  $a$  \_\_\_\_\_  
 $b$  \_\_\_\_\_  
 $\angle B$  \_\_\_\_\_

3.  $\sin 45 = \frac{1}{\sqrt{2}}$

$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 45 =$  \_\_\_\_\_

$\cos 60 =$  \_\_\_\_\_

$\tan 45 =$  \_\_\_\_\_

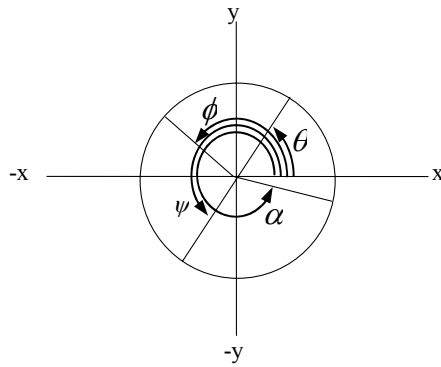
$\sin 30 =$  \_\_\_\_\_

$\sin 0 =$  \_\_\_\_\_

$\tan 60 =$  \_\_\_\_\_

$\cos 0 =$  \_\_\_\_\_

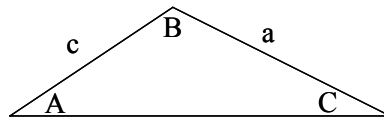
4. Given:



Find: sign of:

$\tan \theta$	_____
$\sin \phi$	_____
$\cos \psi$	_____
$\sin \alpha$	_____

5. Given:



$b$   
 $a = 126$   
 $\angle B = 60^\circ$   
 $\angle C = 65^\circ$

Find:

$\angle A$	_____
$b$	_____
$c$	_____

[Recall the law of sines and/or the law of cosines]

6. Show that:

$$\cos(\alpha + 2\beta) = \cos \beta(\cos \alpha \cos \beta - \sin \alpha \sin \beta) - \sin \beta(\cos \alpha \sin \beta + \sin \alpha \cos \beta)$$

7. Find the corresponding number of radians or degrees

a. 315 degrees

\_\_\_\_\_

b. 120 degrees

\_\_\_\_\_

c. 100 degrees

\_\_\_\_\_

d.  $\pi$  radians

\_\_\_\_\_

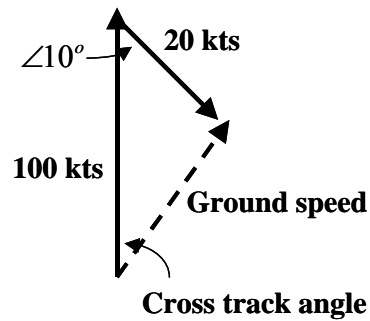
e.  $\frac{5\pi}{4}$  radians

\_\_\_\_\_

f. 1.6 radians

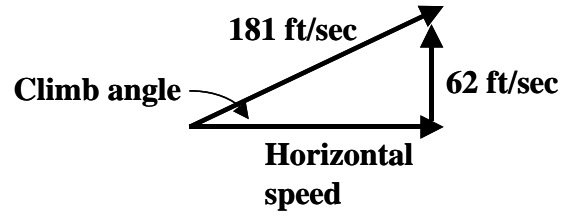
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8. Given an aircraft traveling north at 100 kts into a 20 knot headwind from 350 degrees.



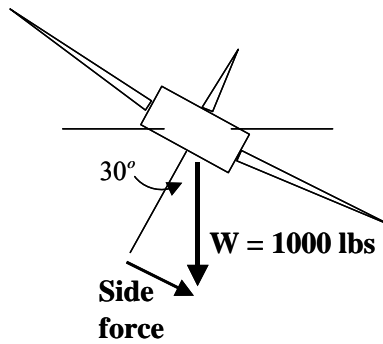
Find: Ground speed and cross track angle

9. Given an aircraft flying at 181 ft/sec and climbing at 62 ft/sec.



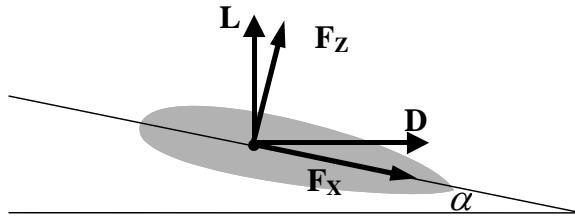
Find: Horizontal speed and climb angle.

10. Given a 1000 lb aircraft in a 30 degree bank.



Find: Side force.

11. Given:



Assuming small angle theory ( $\alpha$  is small), why is  $F_z \approx L$  and  $F_x \approx D$ ?

12. During roll performance testing, the F-99A rolled from 60 degrees left wing down to 60 degrees left wing up in 0.4 seconds.

Find the roll rate in:

Degrees/second \_\_\_\_\_

Radians/second \_\_\_\_\_

13. Plot in the same graph the following trigonometric functions:

a.  $y = \cos\left(\frac{\pi}{2}t\right)$

b.  $y = 3 \cos\left(\frac{\pi}{2}t\right)$

c.  $y = 3 \cos(\pi t)$

d.  $y = 3 \cos\left(\pi t + \frac{\pi}{2}\right)$



What are the periods and frequencies (both in Hz and radians/sec) of those functions?

## Pre-TPS

### Co-ordinate Systems and Graphs Tutorial

- Find the slopes of the lines through the points:
  - (3, 5) and (2, -3)
  - (-1, 2) and (4, -3)
  - (-2, 4) and (-5, -5)
- Find the co-ordinates of a point  $P_1(x, y)$  which is located such that the line  $L_1$  through the origin and  $P_1$  has a slope of +2, and the line  $L_2$  through the point  $P_2(-1, 0)$  and  $P_1$  has a slope of +1.
- Plot the given points. Determine analytically whether or not each group lies on a straight line.
  - $P_1(1, 0)$ ,  $P_2(0, 1)$ ,  $P_3(2, -1)$
  - $P_1(-2, -1)$ ,  $P_2(-1, 1)$ ,  $P_3(1, 5)$ ,  $P_4(2, 7)$
- Given  $P_1(0, -1)$ ,  $P_2(4, 0)$  and  $P_3(3, 4)$ , show that  $P_1P_2P_3$  is a right-angled triangle.
- Find the slope of the straight lines given by:
  - $y = 3x + 5$
  - $x + y = 2$
  - $3x + 4y = 12$
- Find the line that passes through the point (1, 2) and is parallel to the line  $x + 2y = 3$
- Find the equation of the line through the point (1, 4) having a slope of  $60^\circ$
- If A, B, C, and C' are constants, show that
  - the lines  $Ax + By + C = 0$  and  $Ax + By + C' = 0$  are parallel, and that
  - the lines  $Ax + By + C = 0$  and  $Bx - Ay + C' = 0$  are perpendicular
- Given the Fahrenheit (F) versus Celsius (C) curve is a straight line, find its equation given  $C = 0$  when  $F = 32$ , and  $C = 100$  when  $F = 212$ . Find the temperature at which  $C = F$



10. Find the centers and radii of the given circles:

a.  $x^2 + y^2 - 2y = 3$

b.  $x^2 + y^2 + 2x = 8$

c.  $x^2 + y^2 + 2x - 4y + 1 = 0$

11. If V is the vertex and F the focus of a parabola, find the equations of the following parabolas:

a. V (0, 0), F (0, 2)

b. V (-2, 3), F (-2, 4)

c. V (1, -3), F (1, 0)

12. Given a and b are positive, sketch the parabolas:

$$y^2 = 4a^2 - 4ax \text{ and}$$

$$y^2 = 4b^2 + 4bx$$

13. Sketch the following ellipses:

a.  $9x^2 + 4y^2 = 36$  and

b.  $(x - 1)^2/16 + (y + 2)^2/4 = 1$

Logarithms, Radicals and Exponents  
Tutorial

1. Evaluate the following:

a.  $\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2 =$  \_\_\_\_\_

b.  $\frac{a^{10}}{a^4} =$  \_\_\_\_\_

c.  $(a^{n+2})(a^{m+3}) =$  \_\_\_\_\_

d.  $(a^2)^5 =$  \_\_\_\_\_

e.  $(a^{2n})^3 =$  \_\_\_\_\_

f.  $(a^3 + b^5)^0 =$  \_\_\_\_\_

2. Evaluate the following:

a.  $4^{\frac{2}{3}}$  \_\_\_\_\_

b.  $8^{-\frac{2}{3}}$  \_\_\_\_\_

c.  $(x^5)^{-4}$  \_\_\_\_\_

d.  $\left(a^{\frac{2}{3}}\right)^{\frac{3}{4}}$  \_\_\_\_\_

e. Expand  $\sqrt[7]{x^2y^5}$  \_\_\_\_\_

f. Simplify  $\sqrt{27}$  \_\_\_\_\_

3. Write the following in logarithmic form:

a.  $7^2 = 49$

\_\_\_\_\_

b.  $3^3 = 27$

\_\_\_\_\_

c.  $2^{-3} = \frac{1}{8}$

\_\_\_\_\_

d.  $\sqrt[3]{8} = 2$

\_\_\_\_\_

4. Write the following in exponential form:

a.  $\log_3 81 = 4$

\_\_\_\_\_

b.  $\log_9 27 = \frac{3}{2}$

\_\_\_\_\_

c.  $\log_{10} 50 = 1.699$

\_\_\_\_\_

5. Simplify the following:

$\log_{10}(5)(9) + \log_{10} \frac{25}{9} - \log_{10} 5$

\_\_\_\_\_

6. Find:

a.  $\log_{10} 3860$

\_\_\_\_\_

b.  $\log_{10} 5.46$

\_\_\_\_\_

c.  $\log_{10} .00235$

\_\_\_\_\_

d.  $\log_{10} .0000129$

\_\_\_\_\_

e.  $\log_{10} 72800$

\_\_\_\_\_

7. Solve for x:

a.  $(3)(10^x) = 27$

\_\_\_\_\_

b.  $e^{2(x-5)} = 30$

\_\_\_\_\_

c.  $2e^x = 8$

\_\_\_\_\_

d.  $\ln x - \ln (x-1) = 2$

\_\_\_\_\_

Complex Numbers  
**Tutorial**

1. Perform the indicated operations:

a.  $(3 - 4i) - (-5 + 7i)$  \_\_\_\_\_

b.  $(4 + 2i) + (-1 + 3i)$  \_\_\_\_\_

c.  $(2 + i)(3 + 2i)$  \_\_\_\_\_

d.  $(3 - 4i)(3 + 4i)$  \_\_\_\_\_

e.  $\frac{1+3i}{2-i}$  \_\_\_\_\_

f.  $\frac{3-2i}{2+3i}$  \_\_\_\_\_

2. Find the conjugate of the following:

a.  $2 + I$  \_\_\_\_\_

b.  $2 - 3i$  \_\_\_\_\_

c.  $-4 + 2i$  \_\_\_\_\_

d.  $-4 - 3i$  \_\_\_\_\_

e.  $3i - 7$  \_\_\_\_\_

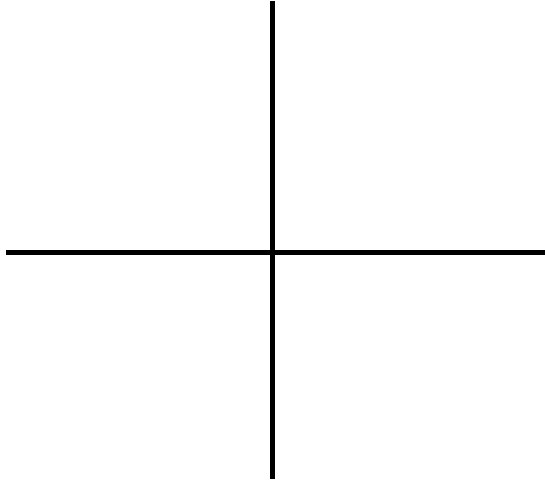
3. Graph the following:

a.  $3 + 2i$

b.  $2 + i$

c.  $-2 - i$

d.  $-1 + 3i$

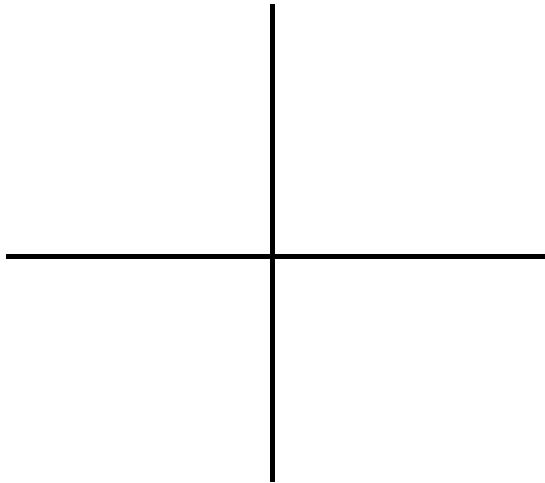


4. Graphically add the following:

a.  $(3 + 4i) + (2 - 3i)$

b.  $(3 - 4i) + (2 + i)$

c.  $(-3 + 3i) - (2 + i)$



5. Express the following in polar form:

a.  $+1+i\sqrt{3}$

\_\_\_\_\_

b.  $6\sqrt{3}+6i$

\_\_\_\_\_

c.  $0+4i$

\_\_\_\_\_

d.  $-1+i$

\_\_\_\_\_

6. Express the following in rectangular form:

a.  $4(\cos 60^\circ + i \sin 60^\circ)$

\_\_\_\_\_

b.  $3(\cos 90^\circ + i \sin 90^\circ)$

\_\_\_\_\_

c.  $2(\cos 45^\circ + i \sin 45^\circ)$

\_\_\_\_\_

7. Use De Moivre's theorem to evaluate the following and express results in  $a + bi$  form:

a.  $(1 + \sqrt{3}i)^5$  \_\_\_\_\_

b.  $\sqrt{(1 - \sqrt{3}i)}$  \_\_\_\_\_

8. Express the following in the alternate forms requested:

a.  $4(\cos 60^\circ + i \sin 60^\circ)$  exponential form: \_\_\_\_\_

b.  $6\sqrt{3} + 6i$  exponential form: \_\_\_\_\_

e.  $4e^{\frac{\pi}{2}i}$  polar form: \_\_\_\_\_

rectangular form: \_\_\_\_\_



## Determinant & Matrix Tutorial

1. Solve the following determinants:

a.  $\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$

b.  $\begin{vmatrix} -3 & -4 \\ 2 & 7 \end{vmatrix}$

c.  $\begin{vmatrix} 1 & 1 & 1 \\ 3 & -3 & -3 \\ 1 & -1 & 2 \end{vmatrix}$

d.  $\begin{vmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{vmatrix}$

2. Solve the following using Cramer's Rule

$$x + y + z = 0$$

$$3x - 3y - 3z = 12$$

$$x - y + 2z = -7$$

3. Add or subtract the following matrices

a.  $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -3 & -4 \\ 2 & 7 \end{bmatrix}$

b.  $\begin{bmatrix} 1 & 1 & 1 \\ 3 & -3 & -3 \\ 1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 1 \\ 5 & 6 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

4. Multiply the following

$$3 \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 6 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

5. Transpose

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \\ 7 & 5 \end{bmatrix}$$

6. Write the following set of equations in matrix form

$$\begin{aligned} 2x + 7y &= 26 \\ 5x - 2y &= 14 \end{aligned}$$

7. Solve for x, y, and z [Hint: Cramer]

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

## Vector Algebra Tutorial

1. Is  $\bar{V} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$  a unit vector? Why?

2. Find the unit vector for  $\bar{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

3. Are the following two vectors equal? Why?

$$\bar{A} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\bar{B} = 4\hat{i} + 6\hat{j} - 2\hat{k}$$

4. Given:

$$\bar{F}_1 = 2\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\bar{F}_2 = -5\hat{i} + \hat{j} + 3\hat{k}$$

$$\bar{F}_3 = \hat{i} - 2\hat{j} + 4\hat{k}$$

$$\bar{F}_4 = 4\hat{i} - 3\hat{j} - 2\hat{k}$$

Add the above. Find the resultant vector and the magnitude of resultant vector.

5. Given:

$$\bar{A} = 3\hat{i} - \hat{j} - 4\hat{k}$$

$$\bar{B} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

$$\bar{C} = \hat{i} + 2\hat{j} - \hat{k}$$

Find a.  $2\bar{A} - \bar{B} + 3\bar{C}$

b.  $|\bar{A} + \bar{B} + \bar{C}|$

6. Given:

$$\bar{A} = 3\hat{i} - \hat{j} - 4\hat{k}$$

$$\bar{B} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

Find a.  $\bar{A} \cdot \bar{B}$

b.  $|\bar{A}| \& |\bar{B}|$

7. Given:

$$\bar{A} = 2\hat{i} + 3\hat{j} - \hat{k}$$

$$\bar{B} = 4\hat{i} + 6\hat{j} - 2\hat{k}$$

Find  $\bar{A} \cdot \bar{B}$  and the angle between  $\bar{A}$  and  $\bar{B}$

8. Evaluate  $\hat{j} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

9. Given:

$$\bar{A} = 3\hat{i} - \hat{j} - 4\hat{k}$$

$$\bar{B} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

Find  $\bar{A} \times \bar{B}$

10. Find the vector perpendicular to  $\bar{A}$  and  $\bar{B}$  below

$$\bar{A} = 2\hat{i} - 6\hat{j} - 3\hat{k}$$

$$\bar{B} = 4\hat{i} + 3\hat{j} - \hat{k}$$

11. Given

$$\bar{A} = 2\hat{i} - 3\hat{j} - \hat{k}$$

$$\bar{B} = \hat{i} + 4\hat{j} - 2\hat{k}$$

Find  $\bar{A} \times \bar{B}$  and  $\bar{B} \times \bar{A}$

12. Evaluate:

a.  $2\hat{j} \times (3\hat{i} - 4\hat{k})$

b.  $(\hat{i} + 2\hat{j}) \times \hat{k}$

**Pre-TPS  
Differentiation  
Tutorial**

1. Using the relationship:

$$\left. \frac{\Delta y}{\Delta x} \right|_{\lim \Delta x \rightarrow 0} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Find the tangent slope of the following:

a.  $f(x) = 2x^2 + x$

b.  $f(x) = 3x^2 + 2x + 1$

2. Using the general rules for differentiating algebraic functions, find the differential for:

a.  $\frac{d}{dx}(2x^2 + x)$

b.  $\frac{d}{dx}(3x^3 - 4x^2 + 5x - 2)$

c.  $\frac{d}{dx}(2u^2v)$  where  $u$  and  $v$  are functions of  $x$

d.  $\frac{d}{dx}\left(\frac{1}{2}x^4 + 5x\right)$

e.  $\frac{d}{dx}(2u^2 / v^3)$  where  $u$  and  $v$  are functions of  $x$

3. If  $y = 2z^2 + z$  and  $z = (x - 2)$

Find  $\frac{dy}{dx}$  using the chain rule

4. Given:  $y = x^3 + 4x + 3$

Find  $\frac{dy}{dx}$

5. Given:  $y = \frac{x^2 - 3}{x + 4}$

Find  $\frac{dy}{dx}$

6. Given:  $y^2 + x - 4 = 0$

Find  $\frac{dy}{dx}$

7. Given:  $x^2 + 2xy - 3y^2 + 11 = 0$

Find  $\frac{dy}{dx}$  and evaluate at the point (2,3)

8. Find  $\frac{d^2y}{dx^2}$  for the following:

a.  $y = 3x^4 - 2x^3 + 6$

b.  $y = 4ax^{1/2}$

c.  $y = (x + 2)(x - 3)$  hint – expand first

d.  $y - x^2 - 12 = x^7 + 3x^4 + 4x^2 - x + 10$

9. Given:  $s = 120t - 16t^2$

Find the velocity,  $ds/dt$ , and the acceleration,  $d^2s/dt^2$

Evaluate the velocity and acceleration at  $t = 2$

10. Find maximum and minimum values for  $x$  and  $y$  given:

a.  $y = x^3 + 2x^2 - 15x - 20$

b.  $y = x^2 - 10$

Sketch both graphs.



**PreTPS  
Integration  
Tutorial**

1. Integrate the following non-definite integrals

a.  $\int (x^3 + 6x^2 + 7) dx$

b.  $\int \frac{dx}{x^2}$

c.  $\int \frac{2x+1}{(x^2+x)} dx$

d.  $\int \sin 3x dx$

e.  $\int (\cos 4x + \sec^2 x) dx$

f.  $\int e^{3x} dx$

2. Evaluate the following definite integrals:

a.  $\int_0^{\frac{\pi}{2}} 3 \sin x dx$

b.  $\int_{-\pi}^{+\pi} 2 \cos x dx$

c.  $\int_0^3 (x^2 + 7x + 6) dx$

3. Integrate by parts:

$$\int x \cdot \sin x dx$$

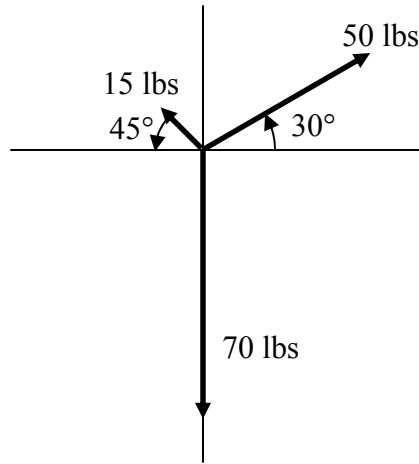
4. Integrate by substitution:

$$\int \sin^3 x \cos x dx$$

5. Find the area under the curve  $y = x^3 + 3x^2 + 2$  between  $x = 0$  and  $x = 2$ .

Statics and Friction  
Tutorial

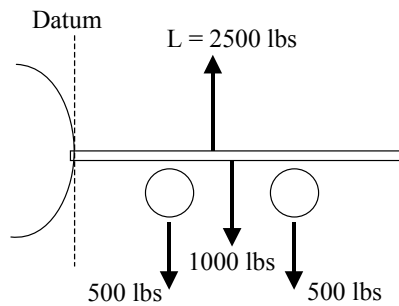
1. Given:



Find the resultant force (magnitude and angle)

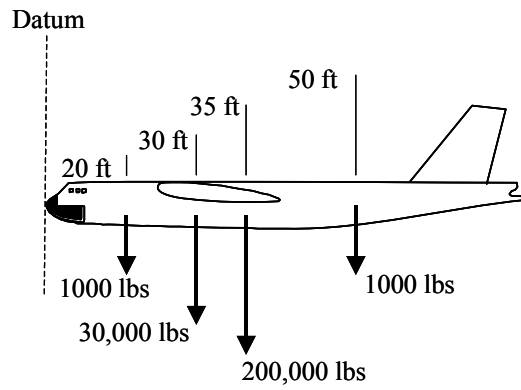
2. Given:

Distance from datum  
Wing center of lift = 15 ft  
Inboard engine = 10 ft  
Outboard engine = 30 ft  
Wing cg = 20 ft



Find:  $\sum M$  around datum

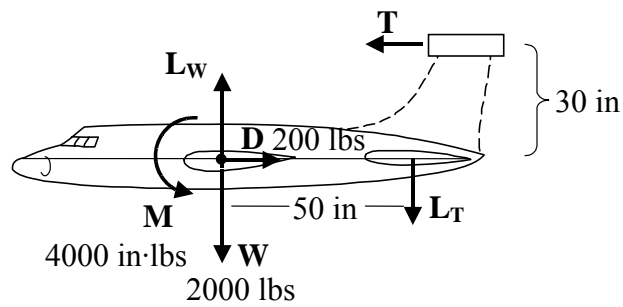
3. Given:



Find: a) Resultant force

b) Distance from datum to resultant force

4. Given:



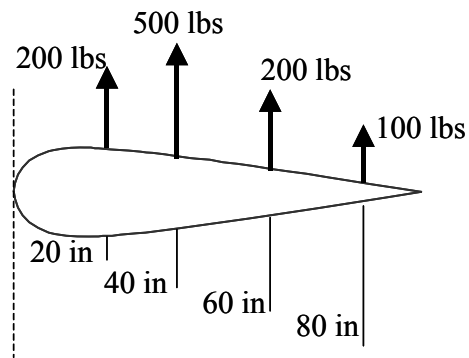
Find the following to keep the aircraft balanced.

a.  $T$

b.  $L_T$

c.  $L_W$

5. Given:



- Find the resultant force ( $F_R$ )
- Find the distance from the leading edge to the resultant force ( $\bar{x}$ ).
- Transfer the resultant force to the 25 inch point and determine the resultant moment .

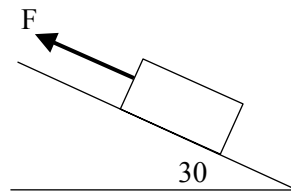
6. Given:



Weight of block = 112 lbs  
 $\mu_s = 0.25$

Find the minimum force required to move the block.

7. Given:

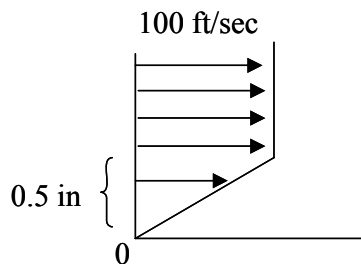


Weight of block = 150 lbs  
 $\mu_s = 0.3$

Find: a) Minimum force required to hold the block at rest.

b) Maximum force required to hold the block at rest.

8. Given:



Calculate  $\frac{dV}{dy}$  from this data

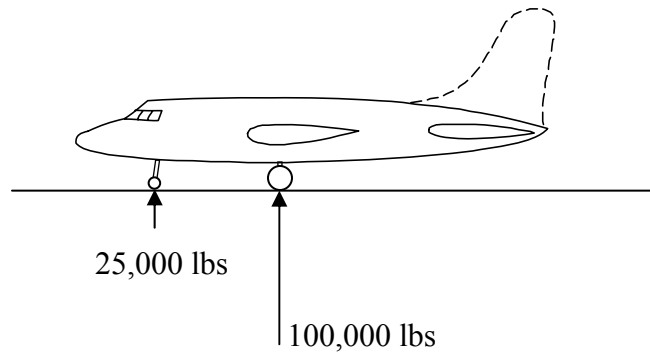
$$\mu = 1.2 \times 10^{-5} \frac{lb \cdot sec}{ft^2}$$

Find: a)  $\frac{dV}{dy}$

b)  $\tau$

c) Shear force acting over a 200  $ft^2$  area

9. Consider an aircraft weighing 125,000 lbs taxiing on the ground.



Assuming that:

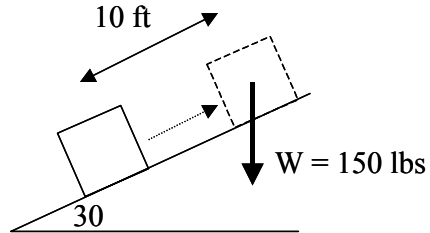
- the reaction force on the nose wheel is 25,000 lbs;
- the reaction force on the main gear is 100,000 lbs (50,000 lbs per wheel)
- the radius of the nose wheel is 25 in;
- the radius of the main gear wheel is 50 in;
- the coefficient of rolling resistance is  $b=1$  in;
- the aerodynamic drag is negligible;

find the engine thrust necessary to maintain a constant ground speed

10. Assuming  $\mu_s=0.4$ , calculate the maximum braking force the crew can apply without skidding.

Work and Energy  
Tutorial

1. Determine the amount of work performed on the block when it is moved 10 ft UP the incline as shown. ASSUME NO FRICTION



2. Determine the amount of work done in #1 above if  $\mu = 0.2$
3. What is the potential energy of a 240,000 lb aircraft flying at 36,000 ft and 380 kts? [(kts)(1.68) = ft/sec]

What is the kinetic energy?

What is the total specific energy?





**Kinematics**  
**Tutorial**

Take  $g$  as  $32 \text{ ft/sec}^2$

1. A train's speed increases uniformly from 30 mi/hr to 60 mi/hr in 5 minutes. Determine the average speed, the distance traveled and the acceleration.

2. A stone dropped from a tower strikes the ground in 3 sec. Determine the height of the tower.

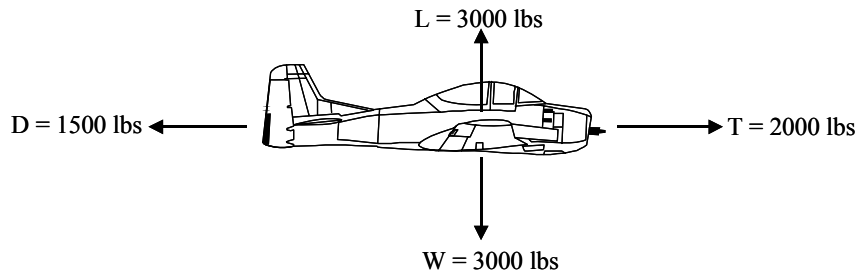
3. A stone is thrown vertically upwards with a velocity of 96 ft/sec. Calculate the time taken to reach the highest point; the greatest height reached; and the total time before the stone hits the ground.

4. A 3 lb body is whirled on a 4 ft string in a horizontal circle. Calculate the tension in the string if the speed is: (a) 8 ft/sec, (b) 2 revolutions per second (RPS).

5. A body rests in a pail which is moved in a vertical circle of radius 2ft. What is the least speed the body must have so as not to fall out when at the top of the path?

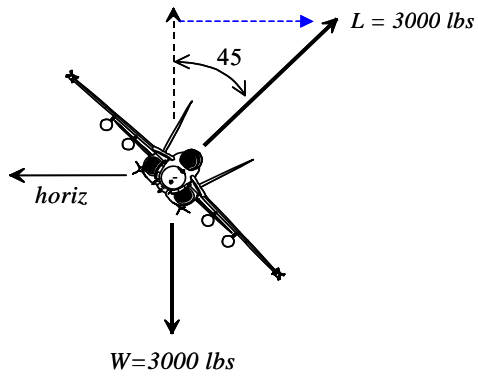
Newton's Laws  
Tutorial

1. Given the following diagram of an aircraft in flight:



- a. Is the aircraft in steady, unaccelerated flight? Why?
  
- b. Calculate the aircraft acceleration

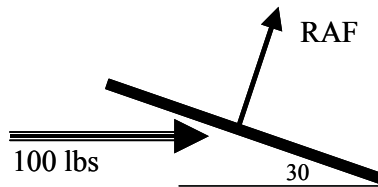
2. Given the following turning aircraft:



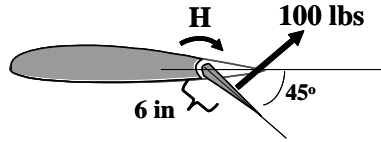
Assume:  $T = D = 2000$  lbs

- a. Is the aircraft in level flight? Why?
  
  
  
  
  
  
  
  
  
  
- b. Calculate the vertical acceleration

3. Given a 100 lb force striking and inclined plate as shown, compute resultant force (RAF).



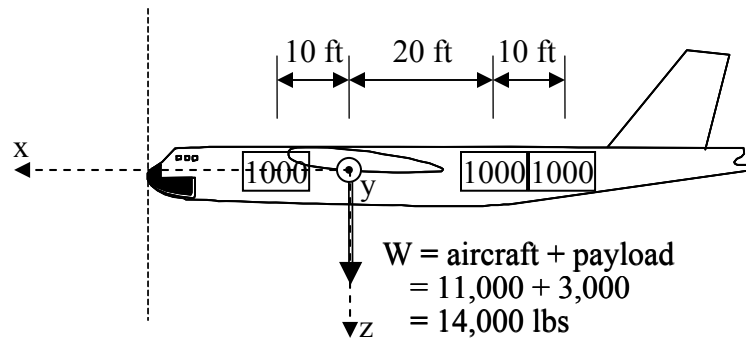
4. In the following diagram a control surface is deflected down  $45^\circ$ . Assume the airflow striking the deflected surface creates the resultant 100 lb force.



- a. In order to hold the deflected surface in place how much hinge moment ( $H$ ) is required?
  
  
  
  
  
  
  
  
  
  
  - b. If the hinge suddenly breaks, what will be the horizontal acceleration of the deflected surface [Assume the deflected surface weights 10 lbs]
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5. An aircraft weighing 20,000 lbs (including payload) drops a 5,000 lb bomb from straight, level unaccelerated flight. Calculate the vertical acceleration of the aircraft immediately after dropping the bomb.

## Inertia Tutorial

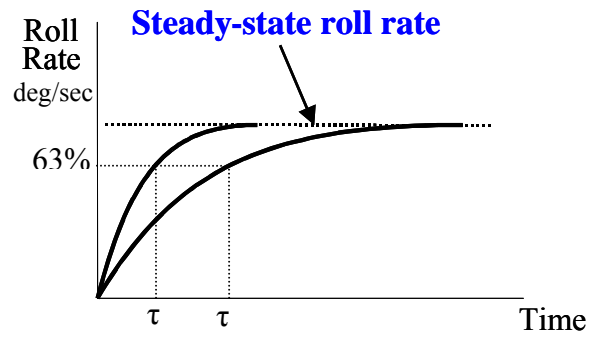
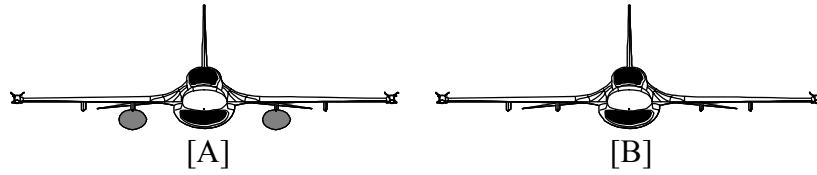
1.



- a. Find  $I_y$  for the aircraft loaded as shown above (empty weight moment of inertia:  $(I_y)_{\text{empty}}=80,000 \text{ slug}\cdot\text{ft}^2$ ).
- b. Find  $I_y$  of the aircraft after the 1000 lb payload is dropped from the forward bay.
- c. If the short period frequency of the aircraft is  $f\left(\frac{1}{I_y}\right)$ , does the short period frequency of the aircraft increase, decrease, or stay the same after the forward payload is released?

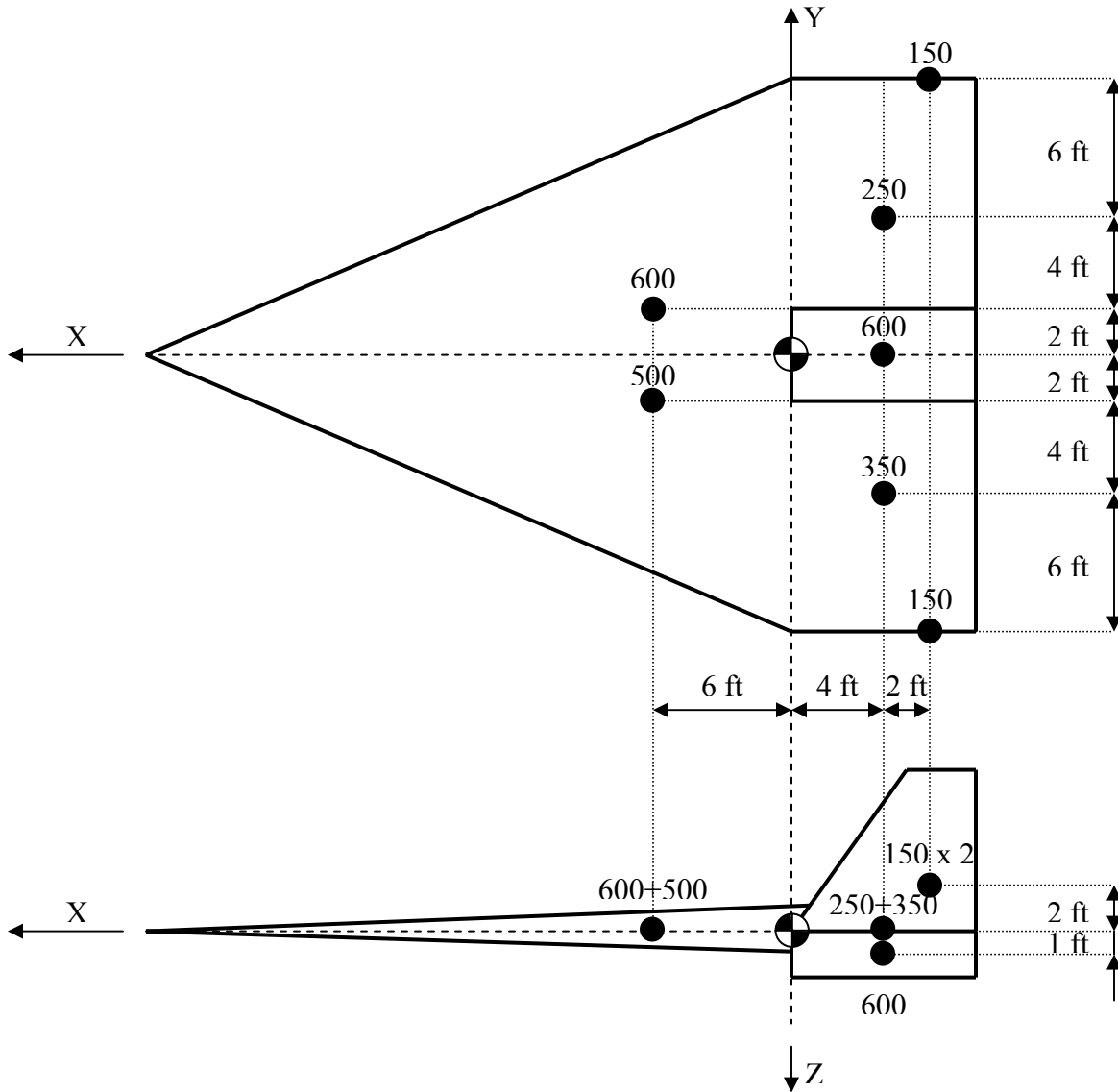
2. The roll mode ( $\tau$ ) time constant is a measure of how quickly the maximum roll rate ( $p$ ) can be reached.

$$\tau = f\left(\frac{I_x}{\text{damping}}\right)$$



Match the roll mode time constant with the appropriate configuration.

3. Given the following future X-airplane, calculate the moments of inertia ( $I_x$ ,  $I_y$ ,  $I_z$ ) and the products of inertia ( $I_{xy}$ ,  $I_{yz}$ ,  $I_{xz}$ ).





**Momentum and Impulse**  
**Tutorial**

1. An 8gm bullet is fired horizontally into a 9kg block of wood which is free to move. The velocity of the block and bullet after impact is 40cm/sec. Calculate the initial velocity of the bullet.

2. A 600lb gun mounted on wheels fires a 10lb projectile with a muzzle velocity of 1800ft/sec at an angle of  $30^\circ$  above the horizontal. Calculate the horizontal recoil velocity of the gun.

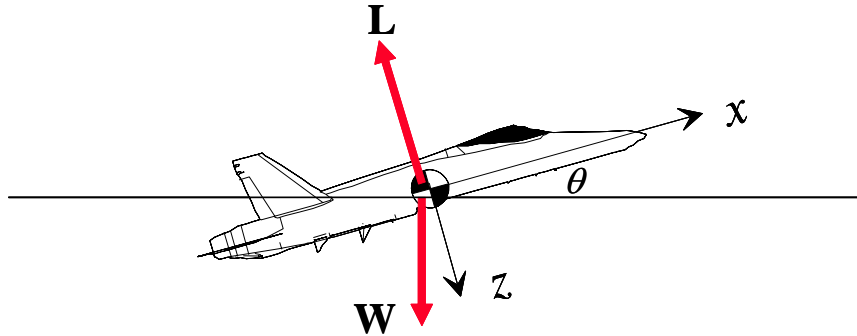
3. Two inelastic masses of 16 and 4 grams move in opposite directions with velocities of 30 and 50 cm/sec respectively. Determine the resultant velocity after impact if they stick together.

4. An 8lb body is acted on by a force for a period of 4 sec during which it gains a velocity of 20ft/sec. Determine the magnitude of the force.

5. A 10-ton locomotive moving at 2ft/sec collides with and is coupled to a 40-ton car at rest on the same straight track. What is their common velocity after impact?

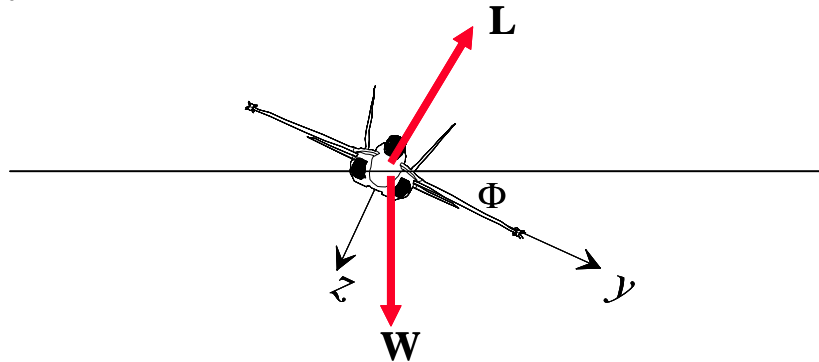
## Axis Transform Tutorial

1. Given the following diagram, resolve  $\mathbf{W}$  onto the body axis and determine the following equations
  - a.  $x_b =$
  - b.  $z_b =$

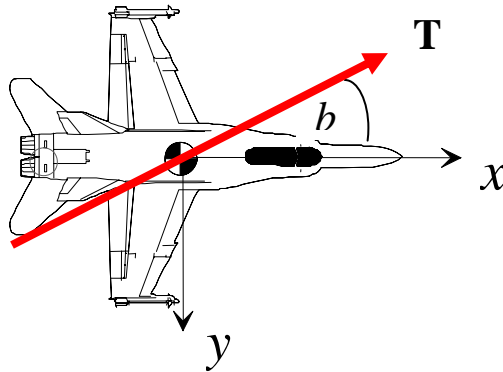
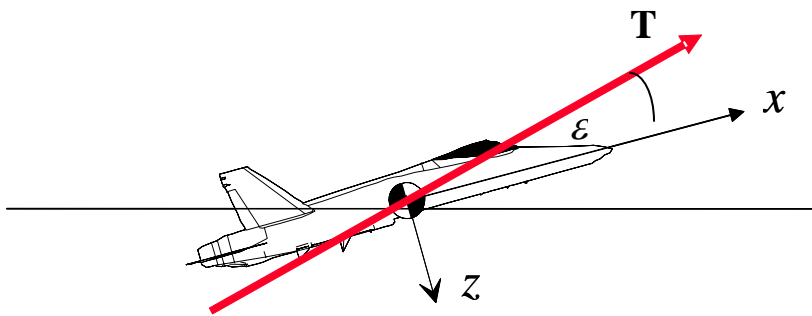


Given the following, resolve  $\mathbf{W}$  onto the body axis and determine the following equations

- a.  $y_b =$
- b.  $z_b =$



2. Transform the thrust onto the body axis and determine the equations for  $x_b, y_b, z_b$



3. Write the following in matrix form

$$x_a = x_b \cos \theta - z_b \sin \theta$$

$$y_a = y_b$$

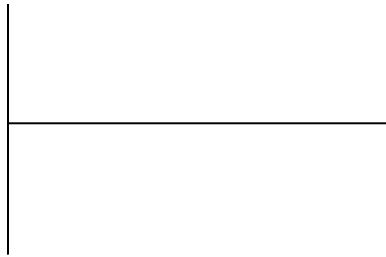
$$z_a = x_b \sin \theta + y_b \cos \theta$$

Motion Analysis  
Tutorial

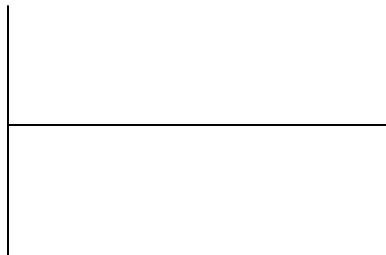
1. Draw a typical trace for the following oscillating system.  
a. Positive damped (stable)



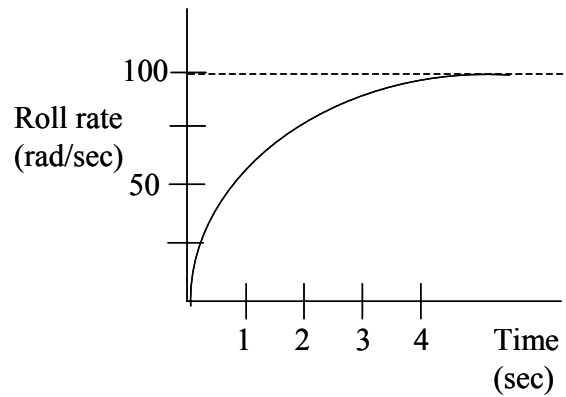
- b. Neutral damped (neutral)



- c. Negative damped (unstable)



2. Given the following 1<sup>st</sup> order response



- Estimate  $\tau$
- Write the time history response equation
- Is the response convergent or divergent?

3. Given the following “s-domain” equations

$$s + .0095 = 0$$

$$s^2 + .875s + 18.4 = 0$$

- Find time constant ( $\tau$ )
- Find natural frequency ( $\omega_n$ )
- Find damping ratio ( $\xi$ )

4. Given the attached trace, calculate the damping ratio ( $\zeta$ ) using the Transient Peak Method.

5. Given the following, calculate the time constant ( $\tau$ ) using

$$\tau = \frac{\Delta t}{\ln\left(\frac{A_1}{A_2}\right)}$$

